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A SIMPLIFIED SCHEME TO PERFORM PRICE ADJUSTING SAMPLING FOR GUA--ETC(U)  
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THESIS

A SIMPLIFIED SCHEME  
TO PERFORM PRICE ADJUSTING SAMPLING  
FOR QUALITY ASSURANCE

by

William Paul Keane

September 1980

Thesis Advisor:

Glenn F. Lindsay

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
	AD A093625	(9)
4. TITLE (and Subtitle)		5. DATE OF REPORT & REMARKS COVERED
A Simplified Scheme to Perform Price Adjusting Sampling for Quality Assurance.		Master's Thesis, September 1980
6. AUTHOR(s)		7. PERFORMING ORG. REPORT NUMBER
William Paul/Keane		
8. PERFORMING ORGANIZATION NAME AND ADDRESS		9. CONTRACT OR GRANT NUMBER(s)
Naval Postgraduate School Monterey, California 93941		
10. CONTROLLING OFFICE NAME AND ADDRESS		11. REPORT DATE
Naval Postgraduate School Monterey, California 93941		September 1980
12. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		13. NUMBER OF PAGES
Naval Postgraduate School Monterey, California 93941		65 pages
		14. SECURITY CLASS. (of this report)
		Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report)		
Approved for public release; distribution unlimited		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number)		
Quality assurance, sampling methods, price adjusting sampling		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number)		
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(Page 1)EDITION OF 1 NOV 68 IS OBSOLETE  
S/N 0102-014-4601UNCLASSIFIED 251450  
SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

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A Simplified Scheme  
to Perform Price Adjusting Sampling  
for Quality Assurance

by

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Submitted in partial fulfillment of the  
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MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

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#### ABSTRACT

This thesis presents a quality assurance scheme which eliminates the necessity of rejecting lots. The mechanism used to motivate suppliers to provide lots at an acceptable quality level is to adjust the price paid for the lot based on the results of a sample. Information contained in Military Standard 105D and a concept referred to as Price Adjusted Single Sampling are combined in order to develop a system which allows a decision maker to find a sampling plan by selecting three parameters.

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## I. INTRODUCTION

The traditional method of specifying an acceptance sampling plan for quality assurance has been to identify the number of items to be inspected, which is referred to as the sample size, and the acceptance number. The acceptance number is defined as the maximum number of unsatisfactory items in the sample which still allows the entire group of items, or lot, to be accepted. If the results of the sampling process indicate that the lot is acceptable, a predetermined fixed price is paid to the supplier for the entire lot. If the lot is judged unacceptable, and therefore rejected, it is returned to the supplier without payment. There are instances, however, when rejecting a lot is not feasible. This thesis examines an alternative method of using the results of the sampling process to determine a variable price to be paid for a lot in consideration for each lot being accepted.

As a result of direction from the Office of Management and Budget, OMB Circular No. A-76 [8], agencies of the federal government have begun preparing for an increase in the amount of commercial and industrial services being performed by private contractors. Many of the services to be contracted are required on a regular basis, such as

janitorial work or bus service. For a service the term item is defined as a scope of work or task. For example, in janitorial work a trash basket is emptied or not emptied, a floor is waxed as required or it is not, or in general, the task is accomplished in a satisfactory or unsatisfactory manner. For bus service an item could be a scheduled stop which would be satisfactory if made within required time limits. Using this definition of an item, the term lot is defined as the number of items which the customer expects to receive over a given period of time, e.g., the number of emptied trash baskets or the number of scheduled stops in a month. From a practical viewpoint, unacceptable lots of this kind cannot be rejected and returned to the contractor. Military Standard 105D furnishes a wide range of sampling plans for general use; however, in this context the standard does not appear to be adequate or appropriate. An alternative sampling scheme directed towards determining how much to pay for a lot needs to be developed. A possible scheme will be presented in this paper.

In the next chapter we will develop the idea of expected price paid per satisfactory item and examine the relationship of this price to sampling plans in Military Standard 105D.

Price Adjusted Single Sampling (PASS) was originally introduced by Joseph Foster [4] and subsequently developed by

Foster and Oscar Perry [5] and [6]. It is a generalized procedure to find a sampling plan consisting of a sample size and a formula to compute payments based on the number of unacceptable items found. In Chapter 3 the mathematics of this procedure will be reviewed in detail with particular attention being given to those items critical to the development of the proposed sampling scheme.

In later chapters equations necessary to create a simplified sampling scheme are developed. Then using an arbitrary set of parameters and attempting to duplicate the simplicity of the Military Standard, a model scheme is constructed. A discussion of the sensitivity of the scheme to changes in the parameters is followed by a set of recommendations for further study.

## II. EXPECTED COST PER SATISFACTORY ITEM

Expected cost per satisfactory item,  $C(p)$ , is an important idea utilized in creating the sampling scheme presented in this paper. After a brief review of Military Standard 105D, it will be shown that Military Standard 105D exhibits an interesting relationship between  $C(p)$  and acceptable quality level, AQL.

### A. MILITARY STANDARD 105D

Military Standard 105D has been a widely used set of sampling plans since being issued in 1963. In order to use Military Standard 105D, it is necessary to know lot size,  $N$ , and to have designated an "acceptable quality level," or AQL. The AQL is set by the customer and is defined as the percent of defective items (in a lot) which the customer considers acceptable. Under plans from Military Standard 105D, lots having AQL quality will be accepted with a high probability. It is also necessary to decide on an "inspection level" which determines the relationship between lot size and sample size.

Although Military Standard 105D provides sets of double and multiple sampling plans, we shall be primarily concerned with single sampling. Each single sampling plan in the standard provides a sample size,  $n$ , and an acceptance

number,  $A_c$ , depending on the type of inspection in effect, either normal, tightened, or reduced. Criteria are provided in the plan to shift from one type of inspection to another, depending on the experience with previously inspected lots. These shifts will result in the values of  $A_c$  and/or  $n$  being changed.

It has been stated that "Military Standard 105D plans are not based upon cost concepts" [1]. This is, at least, an arguable point since the rules for using the standard allow the consumer to adjust the amount of sampling done per lot in relation to lot size by choosing a level of inspection consistent with cost considerations. However, by another criterion, Military Standard 105D does involve cost, i.e., expected cost per satisfactory item.

#### B. EXPECTED COST PER SATISFACTORY ITEM

Expected cost per satisfactory item,  $C(p)$ , is defined as follows:

$$C(p) = \frac{\text{Pr}(\text{accept lot} | p) NC'}{N(1-p)} = \frac{P_a C'}{1-p} . \quad (1)$$

Here  $C'$  is the contracted price paid for each item in an accepted lot.

To illustrate the relationship between  $C(p)$  and  $p$ , data was taken from the table for normal inspection contained in Military Standard 105D. The AQL's 1.0, 1.5, 2.5, 4.0, 6.5, and 10.0 were chosen as the area of interest. In order to

use available tables of cumulative binomial probabilities [7], sample size was restricted to 50, 80, and 125. Using these tables to determine the probability of acceptance,  $P_a$ , for a given  $p$ , inserting the results into Equation (1), and letting  $C'$  equal 1.0 led to the values shown in Table I.

As can be seen, the value of  $p$  which tends to maximize  $C(p)$  is the AQL. This behavior becomes more evident as AQL increases and as sample size increases.

Since it could be argued that normal inspection is only part of the overall sampling scheme set forth in Military Standard 105D, an analysis was done to determine if shifting among normal, tightened and reduced inspection would have similar results. In order to do this, tables contained in a paper by Gerald Brown and Herbert Rutenmiller [1] were used. These tables yield an acceptance probability,  $P_a$ , based on AQL,  $p$  and  $n$ , which accounts for shifts between types of inspection. For the current investigation values of  $n$  were restricted to the three largest available in the Brown paper for each given AQL. Again letting  $C'$  equal 1.0 and using Equation (1), the values in Table II were computed.

As can be seen by comparing Tables I and II, the results are consistent in that  $C(p)$  is maximized at the AQL. It does not seem likely that this result was an objective when the standard was developed but, rather, is

a fortunate byproduct. It is intuitively appealing that suppliers should be encouraged to provide lots close to AQL quality. This suggests that a sampling scheme could be developed which uses the idea of maximizing  $C(p)$  at the AQL.

TABLE I

C(p), Relationship Between Expected Price Per Satisfactory Item, and Incoming Percent Defective, p, Using Military Standard 105D, Normal Inspection

AQL 1.0%			AQL 1.5%		
<u>p</u>	<u>P<sub>a</sub></u>	<u>C(p)</u>	<u>p</u>	<u>P<sub>a</sub></u>	<u>C(p)</u>
n = 50			n = 50		
0	100.	1.0	0	100.	1.0
.3	99.	.993	1.0	98.6	.996
1.0	91.1	.920	1.7	95.0	.966
2.0	73.6	.751	2.0	92.2	.941
			3.0	81.1	.836
n = 80			n = 80		
0	100.	1.0	0	100.	1.0
.6	99.	.996	2.0	100.	1.020
1.0	95.3	.963	3.0	99.9	1.030
2.0	78.4	.800	4.0	98.8	1.029
			5.0	95.1	1.001
			6.0	86.9	.924
n = 125			n = 125		
0	100.	1.0	0	100.	1.0
.7	99.	.997	1.0	99.8	1.01
1.0	96.3	.973	1.4	99.0	1.004
2.0	75.9	.774	2.0	96.0	.980
			3.0	86.6	.850

TABLE I (Continued)

<u>AQL 2.5%</u>			<u>AQL 4.0%</u>		
<u>p</u>	<u>P<sub>a</sub></u>	<u>C(p)</u>	<u>p</u>	<u>P<sub>a</sub></u>	<u>C(p)</u>
n = 50			n = 50		
1.0	99.9	1.009	3.0	99.6	1.027
1.7	99.0	1.007	3.7	99.	1.028
2.0	98.2	1.002	4.0	98.6	1.027
3.0	93.7	.966	5.0	96.2	1.013
4.0	86.1	.897	6.0	92.2	.981
n = 80			n = 80		
1.0	100	1.010	3.0	99.7	1.028
2.0	99.5	1.015	3.7	99.0	1.028
2.3	99.0	1.013	4.0	98.5	1.026
3.0	96.7	.997	5.0	95.3	1.003
4.0	89.9	.936	6.0	89.3	.950
n = 125			n = 125		
1.0	100	1.010	2.0	100.	1.026
2.0	99.6	1.016	3.0	99.9	1.030
2.3	99.0	1.013	4.0	98.8	1.029
3.0	96.5	.995	5.0	95.1	1.001
4.0	87.1	.907	6.0	86.9	.924

TABLE I (Continued)

<u>AQL 6.5%</u>			<u>AQL 10.0%</u>		
<u>p</u>	<u>P<sub>a</sub></u>	<u>C(p)</u>	<u>p</u>	<u>P<sub>a</sub></u>	<u>C(p)</u>
n = 50			n = 50		
5.0	99.7	1.049	8.0	99.8	1.085
6.0	99.1	1.054	9.0	99.6	1.095
7.0	98.6	1.052	10.0	99.1	1.101
8.0	97.8	1.039	11.0	98.2	1.103
9.0	95.6	1.014	12.0	96.8	1.100
n = 80			n = 80		
4.0	100	1.041	8.0	99.8	1.085
5.0	99.8	1.051	9.0	99.5	1.093
6.0	99.2	1.055	10.0	98.8	1.099
7.0	97.7	1.051	11.0	97.3	1.093
8.0	94.6	1.028	12.0	94.8	1.073
n = 125			n = 125		
4.0	100	1.041	8.0	100	1.087
5.0	99.8	1.051	9.0	99.8	1.097
6.0	99.2	1.055	10.0	99.4	1.104
7.0	97.2	1.045	11.0	98.2	1.104
8.0	92.5	1.005	12.0	95.8	1.088

TABLE II

Relationship Between Expected Price Per Satisfactory Item,  
C(p) and Incoming Percent Defective p, Using Military Standard  
105D and Shifting Types of Inspection

<u>AQL 1.0%</u>			<u>AQL 1.5%</u>		
<u>p</u>	<u>P<sub>a</sub></u>	<u>C(p)</u>	<u>p</u>	<u>P<sub>a</sub></u>	<u>C(p)</u>
n = 50			n = 50		
.25	99.7	.999	.375	100.	1.004
.50	98.2	.987	.750	99.9	1.007
.75	95.3	.960	1.125	99.0	1.001
1.0	90.5	.914	1.5	96.5	.980
1.25	84.3	.854	1.875	92.0	.934
n = 80			n = 80		
.25	100.	1.003	.375	100.	1.004
.50	99.8	1.003	.75	99.9	1.007
.75	99.0	.997	1.125	99.1	1.002
1.0	96.1	.971	1.5	96.9	.984
1.25	90.6	.917	1.875	92.6	.944
n = 125			n = 125		
.25	100.	1.003	.375	100.	1.004
.50	99.9	1.004	.75	100.	1.007
.75	99.0	1.007	1.125	99.8	1.009
1.0	96.3	.973	1.5	98.9	1.004
1.25	91.4	.926	1.875	96.4	.982

TABLE II (Continued)

<u>AQL 2.5%</u>			<u>AQL 4.0%</u>		
<u>p</u>	<u>P<sub>a</sub></u>	<u>C(p)</u>	<u>p</u>	<u>P<sub>a</sub></u>	<u>C(p)</u>
n = 50			n = 32		
.625	100.	1.006	2.0	99.9	1.019
1.25	99.9	1.012	3.0	99.	1.021
1.875	99.	1.009	4.0	96.2	1.002
2.5	96.3	.985	5.0	91.8	.956
3.125	91.4	.943	6.0	82.9	.882
n = 80			n = 50		
.625	100.	1.006	2.0	100.	1.020
1.25	100.	1.013	3.0	99.8	1.029
1.875	99.8	1.017	4.0	98.5	1.026
2.5	98.5	1.010	5.0	95.0	1.000
3.125	95.0	.981	6.0	87.7	.933
n = 125			n = 80		
.625	100.	1.006	2.0	100.	1.020
1.25	100.	1.013	3.0	99.8	1.029
1.875	99.9	1.018	4.0	98.5	1.026
2.5	98.6	1.011	5.0	94.0	.989
3.125	92.7	.957	6.0	83.5	.888

TABLE II (Continued)

<u>AQL 6.5%</u>			<u>AQL 10.0%</u>		
<u>p</u>	<u>P<sub>a</sub></u>	<u>C(p)</u>	<u>p</u>	<u>P<sub>a</sub></u>	<u>C(p)</u>
n = 20			n = 13		
3.25	99.9	1.033	5.0	99.9	1.052
4.875	98.9	1.040	7.5	98.2	1.068
6.5	95.9	1.026	10.0	95.7	1.063
8.125	91.3	.983	12.5	90.1	1.030
9.75	82.1	.910	15.0	82.0	.965
n = 32			n = 20		
3.25	100.	1.034	5.0	100.	1.052
4.875	99.7	1.048	7.5	99.8	1.079
6.5	98.1	1.049	10.0	98.5	1.094
8.125	93.9	1.022	12.5	95.0	1.086
9.75	85.8	.944	15.0	87.7	1.032
n = 50			n = 32		
3.25	100.	1.034	5.0	100.	1.052
4.875	99.8	1.049	7.5	99.8	1.079
6.5	98.4	1.052	10.0	98.4	1.093
8.125	93.4	1.017	12.5	94.0	1.074
9.75	82.2	.911	15.0	83.5	.982

### III. PRICE ADJUSTED SINGLE SAMPLING

One of the purposes of an acceptance sampling program is to motivate the supplier to provide lots possessing at least an acceptable level of quality. This can be done by rejecting lots of apparent poor quality, which increase the supplier's costs, or, as done in Price Adjusted Single Sampling (PASS), by adjusting contract price, which reduces the supplier's receipts.

The underlying concept of price adjusted sampling plans is "economic indifference." In developing his idea of acceptance sampling plans with price differentials [9], Robert Roeloffs recognized this when he stated that "the consumer may be willing to pay a higher price for a product with a lower proportion defective." Roeloffs did not, however, pursue this idea explicitly. Foster and Perry introduced their sampling system using a linear indifference function, subsequently expanding PASS to include a quadratic indifference function, dependent on quality level  $p$ . The function is defined as

$$h(p) = NC(A_0 + A_1 p + A_2 p^2). \quad (2)$$

The constants  $N$  and  $C$  are defined, respectively, as the lot size and the average price per acceptable unit the consumer is willing to pay. Restrictions on the coefficients  $A_0$ ,  $A_1$ , and  $A_2$  will be discussed in Chapter IV.

#### A. STATISTICAL DEVELOPMENT OF PASS

Let us consider a sample of size  $n$  drawn from a lot of  $N$  items. The number of defective items in a sample is denoted by  $x$ . Since the sample is taken without replacement, the random variable  $X$  which represents the number of defective items found is hypergeometrically distributed. It is assumed that  $N$  is much larger than  $n$  so the binomial approximation to the hypergeometric can be used.

To develop a PASS plan, it is necessary to identify a relationship between the total lot price  $P_t$  and  $x$ , the number of defective items in the sample. Let

$$P_t = N(C - g(x)) , \quad (3)$$

where  $g(x)$  is a function of the number of defective items. In order that  $P_t$  decreases as  $x$  increases,  $g(x)$  must be monotone increasing. In Equation (3),  $P_t$  represents the actual amount paid for a lot, while in Equation (1),  $h(p)$  represents what the consumer is willing to pay for a lot if the quality level  $p$  is known. Since the consumer wants to pay in the long run an average amount equal to what he is willing to pay, the expected value of the total lot price is equated to  $h(p)$ ,

$$E[P_t] = E [N(C - g(x))] = h(p). \quad (4)$$

Substituting Equation (2),

$$E [N(C - g(x))] = N C (A_0 + A_1 p + A_2 p^2),$$

and  $NC - NE[g(x)] = NC(A_0 + A_1 p + A_2 p^2),$

and thus:

$$E[g(x)] = C (1 - A_0 + A_1 p + A_2 p^2). \quad (5)$$

In order to use all the information contained in the coefficients of the indifference functions, let

$$g(x) = B_0 + B_1 x + B_2 x^2. \quad (6)$$

Then  $E[g(x)] = B_0 + B_1 E(x) + B_2 E(x^2).$

Since it has been assumed that the binomial approximation applies,

$$E[g(x)] = B_0 + B_1 np + B_2 [np(1-p) + (np)^2]. \quad (7)$$

Equating Equations (5) and (7) and combining terms, we have:

$$C(1-A_0) - CA_1 p - CA_2 p^2 = B_0 + (B_1 + B_2) np + B_2 n(n-1)p^2.$$

Equating coefficients of  $p$  yields:

$$B_0 = C(1 - A_0),$$

$$B_1 = \frac{CA_2 - CA_1(n-1)}{n(n-1)},$$

$$B_2 = \frac{CA_2}{n(1-n)}.$$

Substituting Equation (6) into Equation (3) and using the above relationships,  $P_t$  becomes:

$$P_t = NC \left[ 1 - (1-A_0 + \frac{A_2x}{n(n-1)} - \frac{A_1(n-1)x}{n(n-1)} + \frac{A_2x^2}{n(1-n)}) \right],$$

or  $P_t = NC \left( A_0 + \frac{A_1x}{n} - \frac{A_2x}{n(n-1)} + \frac{A_2x^2}{n(n-1)} \right) = NCQ(x) \quad (8)$

This is the final formula for computing payments.

In order to completely specify the plan, sample size,  $n$ , must be known. To find  $n$  it is necessary to define a PASS relationship involving producer's or consumer's risk.

#### B. PRODUCER'S AND CONSUMER'S RISK

Foster and Perry define the relationship involving producer's risk as follows: If the fraction defective is some predetermined quality level  $p$ , then the probability that the price paid per nondefective item is less than or equal to a specified lower bound,  $L$ , should be less than or equal to a predetermined probability,  $\alpha$ . That is,

$$\Pr \{ \text{Price per nondefective item} \leq L \mid p = p_1 \} \leq \alpha. \quad (9)$$

The predetermined probability of being paid less than a specified lower bound  $\alpha$  is the producer's risk. This definition can be contrasted to that given by Acheson Duncan [2], who defines "producer's risk is the probability of rejection....It is usually used in reference to rejection of lots from a process the average quality of which equals the AQL."

The relationship involving consumer's risk,  $\beta$ , for some specified upper bound,  $U$ , can be written:

$$\Pr \{ \text{Price per nondefective item} \geq U \mid p = p_2 \} \leq \beta. \quad (10)$$

Consumer's risk is the probability that the consumer will pay more than a specified upper bound,  $U$ , if the lot quality is a specified fraction defective. Historically, consumer's risk has been defined as "the probability of accepting a lot the quality of which is equal to (a specified) fraction defective." [2]

In lieu of discussing producer or consumer risk, it is also possible to refer to producer or consumer protection. If producer's risk is the probability  $\alpha$  that the price received will be less than some lower bound,  $L$ , then producer protection,  $1-\alpha$ , is the probability that the price will be greater than or equal to  $L$ . Consumer protection is similarly defined.

Equations (9) and (10) can be rewritten as follows:

$$\Pr \{ NCQ(x) \leq NL(1-p_1) \} \leq \alpha,$$

$$\text{or} \quad \Pr \left\{ Q(x) - \frac{L}{C}(1-p_1) \leq 0 \right\} \leq \alpha, \quad (11)$$

$$\text{and} \quad \Pr \left\{ Q(x) - \frac{U}{C}(1-p_2) \geq 0 \right\} \leq \beta, \quad (12)$$

$$\text{where} \quad Q(x) = A + \frac{A_1 x}{n} - \frac{A_2 x}{n(n-1)} + \frac{A_2 x^2}{n(n-1)}.$$

The problem now is to find the minimum value of  $n$  which will result in either Equation (11) or (12) being satisfied, depending upon whether producer or consumer

protection is desired. Consider the case where  $h(p)$  is concave, i.e.,  $A_2$  is negative. The procedure is one of successive approximation. Referring to Equation (11), if we fix  $n$ , we could find an  $x$  such that:

$$A + \frac{A_1 x}{n} - \frac{A_2 x}{n(n-1)} + \frac{A_2 x^2}{n(n-1)} - \frac{L}{C} (1-p_1) = 0. \quad (13)$$

Let  $x_1$  and  $x_2$  be the roots of this equation:  $x_1 \leq x_2$ .

Here,  $\Pr (Q(x) - \frac{L}{C} (1-p_1) \leq 0) \leq \alpha$  is equivalent to

$$\Pr (X \leq x_1) + \Pr (X \geq x_2) \leq \alpha. \quad (14)$$

Similarly, referring to Equation (12) and assuming  $A_2$  to be negative.

$$\Pr (Q(x) - \frac{U}{C} (1-p_2) \geq 0) = \Pr (x_1 \leq X \leq x_2) \leq \beta. \quad (15)$$

To find the minimum sample size, the solution procedure is to assume an  $n$ , then solve for  $x_1$  and  $x_2$ . Knowing  $n$ ,  $x_1$ , and  $x_2$ , one may calculate the probability that the random variable  $X$  is in the region specified by Equation (14) or (15), whichever is applicable. We continue incrementing  $n$  until the probability of occurrence is less than or equal to the desired value.

Similar results can be obtained if  $L(p)$  is convex. However, since the convex case does not allow the cost per satisfactory item to be maximized at a quality level other than zero, it is not considered further.

After the required sample size for PASS is established, and given that the indifference function is known, solving for the coefficients of the formula for computing payments, Equation (8), follows routinely. However, as Foster and Perry have pointed out, identifying the indifference function requires considerable effort, and finding the sample size requires a time consuming trial and error process. PASS could be much more useful if it could be converted into a set of tables in a format similar to Military Standard 105D.

#### IV. A MODEL FOR A PASS SCHEME

As stated previously, the process of developing an indifference function  $h(p)$  can be time consuming. This chapter will propose several ideas which, when combined, simplify the selection of PASS coefficients  $A_0$ ,  $A_1$ , and  $A_2$ . Prior to doing this, however, a set of restrictions on these coefficients will be identified.

##### A. RESTRICTIONS ON THE INDIFFERENCE FUNCTION COEFFICIENTS

In Chapter III we considered  $h(p)$  to be a concave function, thus establishing the first restriction that  $A_2$  be less than or equal to 0.

Recalling that  $h(p) = NC(A_0 + A_1 p + A_2 p^2)$  represents what the customer is willing to pay for a lot having known quality  $p$ , it is unlikely that he would be willing to pay more than  $NC$  if the percent defective were zero. It is also unlikely that the supplier would accept less than  $NC$  if  $p = 0$ . Therefore, the second restriction is that  $A_0 = 1.0$ .

Another restriction on the coefficients can be traced to the function  $g(x) = \beta_0 + \beta_1 x + \beta_2 x^2$ , which in terms of the  $A_i$ 's is

$$g(x) = P(1-A_0 + (\frac{A_2}{n-1} - A_1) \frac{x}{n} + \frac{A_2}{n(1-n)} x^2).$$

In order for  $g(x)$  to be monotone increasing, as required in Chapter III,  $A_1$  must be less than or equal to 0.

Two other restrictions appear to be implicit in the indifference function itself. The value of  $h(p)$  should be less than or equal to 0 when  $p$  equals 1.0. Therefore,  $A_1 + A_2$  must be less than or equal to -1.0. If we require that  $h(p)$  cannot be less than 0 if  $p \leq 0.5$ , then  $A_2 + 2 A_1 \geq -4$ . Table III summarizes these restrictions.

TABLE III  
Restrictions on Indifference Curve Coefficients

1.)	$A_2 \leq 0$	necessary
2.)	$A_0 = 1$	not mandatory
3.)	$A_1 \leq 0$	necessary
4.)	$A_1 + A_2 \leq -1$	not mandatory
5.)	$-4 \leq A_2 + 2 A_1$	not mandatory

It may be possible, on an individual case basis, to develop an indifference function having coefficients which violate these restrictions. Convex functions are an example. In this thesis, however, our subsequent work will deal only with functions which comply with the conditions shown in Table III.

#### B. THE PROPOSED MODEL

As stated in Chapter III, the first step in formulating a PASS plan is to identify the indifference function. When

a quadratic function is assumed, the problem then becomes one of identifying the coefficients  $A_0$ ,  $A_1$ , and  $A_2$ . In the previous section it was noted that  $A_0$  should be set equal to 1.0. Therefore, we might seek two equations in two unknowns which could be solved to determine  $A_1$  and  $A_2$ . Recalling that  $C(p)$  is defined as the expected price paid per satisfactory item, and using the relationships developed in the PASS system, we have

$$C(p) = E\left[\frac{P_t}{N(1-p)}\right] = \frac{E(P_t)}{N(1-p)} = \frac{h(p)}{N(1-p)}; p \neq 1$$

$$\text{or } C(p) = \frac{C(1 + A_1p + A_2p^2)}{(1-p)} \quad (16)$$

By taking the first derivative of Equation (16) and setting it equal to zero, a local extremum can be found. Thus

$$\frac{d(C(p))}{dp} = C \left( \frac{(1-p)(A_1 + 2A_2p) + (1 + A_1p + A_2p^2)}{(1-p)^2} \right) = 0,$$

which implies that  $1 + A_1 + A_2(2p - p^2) = 0$ .

Since the extremum (maximum price paid) should occur when  $p$  is equal to acceptable quality level, this equation becomes

$$1 + A_1 + A_2(2 AQL - AQL^2) = 0. \quad (17)$$

Another equation which can be used is the indifference function. Let us consider the quality level  $p = p_0$  to be

the level at which  $h(p) = 0$ , or  $NC (1 + A_1 p_0 + A_2 p_0) = 0$

$$\text{which gives } (1 + A_1 p_0 + A_2 p_0^2) = 0. \quad (18)$$

Solving for  $A_1$  in Equation (17) and inserting this into (18) yields  $1 + p_0 (A_2 (AQL^2 - 2 AQL) - 1) + A_2 p_0 = 0$ ,

$$\text{or } A_2 = \frac{p_0 - 1}{p_0 (AQL^2 - 2 AQL) + p_0^2}, \quad (19)$$

$$\text{and subsequently } A_1 = \frac{(1 - p_0) (2 AQL - AQL^2)}{p_0 (AQL^2 - 2 AQL) + p_0^2} - 1. \quad (20)$$

It is now possible to create a table of  $A_1$  and  $A_2$  values for a given set of AQL's and  $p_0$ 's. Such a table would permit one to determine  $A_1$  and  $A_2$  for specified values of the acceptable quality level and the fraction defective at which the customer is unwilling to pay anything for the product received. For the purpose of developing a table using the proposed model, AQL's were chosen to be 0.03, 0.05, 0.08, 0.10, 0.15, 0.20, 0.25, and 0.30. Values for  $p_0$  were taken to be 0.5, 0.6, 0.7, 0.8, 0.9, and 1.0. Using these values in Equations (19) and (2) resulted in Table IV.

These are two particularly interesting aspects of Table IV. The first is that when  $p_0 = 1.0$ , the values of the coefficients are not affected by changing the AQL. Thus, in order to meet the criteria for the model, the indifference function must be linear when  $p_0 = 1.0$ . A second observation is that the coefficients which would be in the

lower left corner of Table IV do not meet the restrictions identified in Table III. Thus, these values cannot be used and have been deleted. This leaves a set of AQL's and  $P_0$ 's excluded from the proposed sampling scheme. However, this situation is not necessarily a problem, since the excluded sets are those which would not be expected to be used. For example, it is difficult to imagine a situation in which a customer would be willing to pay the maximum price per non-defective item at an AQL = .3 and at the same time specify a  $p_0 = 0.5$  at which he is unwilling to pay anything.

Having developed a set of coefficients for the indifference function, the next step is to consider sample size. The next chapter will discuss the sensitivity of sample size to PASS parameter changes in general and describe a method to find a sample size for the set of indifference functions whose coefficients are given in Table IV.

TABLE IV

Values for Indifference Function Coefficients  $A_1$  and  $A_2$

<u>AQL</u> <sup>(1)</sup>	$p_0$ <sup>(2)</sup>		.5	.6	.7	.8	.9	1.0
	$A_1$	$A_2$						
.03	$A_1$		- .866	- .927	- .960	-.980	- .992	-1.000
	$A_2$		-2.208	-1.233	- .669	-.337	- .132	0
.05	$A_1$		- .758	- .871	- .931	-.965	- .987	-1.000
	$A_2$		-2.484	-1.327	- .711	-.356	- .138	0
.08	$A_1$		- .557	- .771	- .880	-.941	- .977	-1.000
	$A_2$		-2.887	-1.493	- .784	-.387	- .149	0
.10	$A_1$		- .387	- .691	- .840	-.922	- .970	-1.000
	$A_2$		-3.226	-1.626	- .840	-.410	- .156	0
.15	$A_1$	(3)	- .426	- .719	-.867	- .950	-1.000	
	$A_2$	(3)	-2.067	-1.014	-.478	- .179	0	
.20	$A_1$	(3)	0	- .546	-.795	- .926	-1.000	
	$A_2$	(3)	-2.778	-1.261	-.568	- .206	0	
.25	$A_1$	(3)	(3)	- .286	-.698	- .895	-1.000	
	$A_2$	(3)	(3)	-1.633	-.690	- .240	0	
.30	$A_1$	(3)	(3)	(3)	-.560	- .855	-1.000	
	$A_2$	(3)	(3)	(3)	-.862	- .285	0	

(1) AQL = Acceptable Quality Level

(2)  $p_0$  = Fraction defective at which the consumer is unwilling to pay anything.

(3) Values calculated do not satisfy the restrictions given in Table III.

## V. VARIOUS CHARACTERISTICS OF THE PROPOSED SAMPLING SCHEME

In this chapter we will examine the general PASS system for sensitivity to changes in its parameters and also investigate the proposed scheme in this regard. Then we will look at the proposed scheme to see if it lends itself to a tabular format similar to that of Military Standard 105D.

### A. SENSITIVITY OF THE GENERAL PASS MODEL

In examining the sensitivity of the PASS model, the question which needs to be answered is "How does the selection of parameters, such as  $A_0$ ,  $A_1$ ,  $A_2$ , and  $p$ , affect the required sample size,  $n$ . The critical relationship to determine the sensitivity of sample size to these parameters is Equation (13),

$$A_0 + \frac{A_1 x}{n} - \frac{A_2 x}{n(n-1)} + \frac{A_2 x^2}{n(n-1)} - \frac{L}{C} (1-p_1) = 0.$$

If  $A_0$ ,  $A_1$ , and  $A_2$  comply with the restrictions given in Chapter IV, then there is only one root of Equation (13) which is of interest. The smaller root,  $x_1$ , is negative and of no relevance in Equation (14). So we have

$$x_2 = \left[ \frac{A_2}{n(n-1)} - \frac{A_1}{n} - \sqrt{\left( \frac{A_1}{n} - \frac{A_2}{n(n-1)} \right)^2 - 4 \left( A_0 - \frac{L}{C} (1-p_1) \right) \frac{A_2}{n(n-1)}} \right] / \frac{2A_2}{n(n-1)}. \quad (21)$$

As explained in Chapter III, the procedure for finding the minimum sample size is as follows. Fix  $n$  and calculate

the value of  $x_2$ . Then  $\Pr(X \geq x_2)$  is evaluated, using the cumulative binomial distribution, and compared to  $\alpha$ . If  $\Pr(X \geq x_2) > \alpha$ , then the sample size is increased which results in a larger  $x_2$ . This is continued until an  $n$  is found such that  $\Pr(X \geq x_2) \leq \alpha$ . For a given sample size,  $n$ , it is important to see how changes in  $A_1$ ,  $A_2$ , and  $Z$  affect  $x_2$ , where  $Z$  is defined as  $Z = \frac{L}{C} (1-p_1)$ . Taking the partial differentials of Equation (21) results in the following equations:

$$\frac{\partial x_2}{\partial A_1} = \frac{1-n}{2A_2} \left( 1 + \left[ \left( \frac{A_1}{n} - \frac{A_2}{n(n-1)} \right)^2 - 4(1-Z) \frac{A_2}{n(n-1)} \right]^{-\frac{1}{2}} \left( \frac{A_1}{n} - \frac{A_2}{n(n-1)} \right) \right),$$

$$\frac{\partial x_2}{\partial A_2} = \frac{A_1(n-1)}{2A_2^2} - \left[ \frac{A_2}{n(n-1)} \left[ \left( \frac{A_1}{n} - \frac{A_2}{n(n-1)} \right)^2 - 4(1-Z) \frac{A_2}{n(n-1)} \right]^{-\frac{1}{2}} \right.$$

$$\left. \left( \frac{-2}{n(n-1)} \left( \frac{A_1}{n} - \frac{A_2}{n(n-1)} \right) \left( \frac{-4(1-Z)}{n(n-1)} \right) \right) - \left[ \left( \frac{A_1}{n} - \frac{A_2}{n(n-1)} \right)^2 - 4(1-Z) \frac{A_2}{n(n-1)} \right]^{-\frac{1}{2}} \right.$$

$$\left. \left( \frac{2}{n(n-1)} \right) \right] / \left( \frac{2A_2}{n(n-1)} \right)^2,$$

$$\text{and } \frac{\partial x_2}{\partial Z} = \frac{n(1-n)}{A_2} \left[ \left( \frac{A_1}{n} - \frac{A_2}{n(n-1)} \right)^2 - 4(1-Z) \frac{A_2}{n(n-1)} \right]^{-\frac{1}{2}} \frac{A_2}{n(n-1)}.$$

Although these expressions are very unwieldy, they indicate the following:  $x_2$  increases as  $A_1$  or  $A_2$  increases. As  $Z$  decreases,  $x_2$  increases, which implies that  $x_2$  increases as the ratio  $\frac{L}{C}$  decreases or as  $p_1$  increases. It should be noted that in Table IV as the values of  $A_1$  decrease across a row, the corresponding values of  $A_2$  increase. This behavior makes the finding that the value of  $x_2$  is directly related

to  $A_1$  and  $A_2$  particularly important, since it may be found that for a given  $p_0$ , the minimum sample size may depend only on the  $\frac{L}{C}$  ratio and not on AQL.

#### B. EXAMINING THE PROPOSED MODEL

In order to examine the sensitivity of the proposed scheme, a computer program was written to determine the values of  $x_2$  for each set of the indifference function coefficients given in Table IV as  $\frac{L}{C}$  varied from .9 to 1.0 and as  $n$  varied from 10 to 150. The output of that program is contained in Appendix A. After setting producer's risk at .05, it was possible to consult a table of cumulative binomial probabilities to determine the minimum value of  $x_2$  for a given  $n$  which would result in a probability of occurrence less than or equal to .05. Table V contains these values of  $x_2$ . Having this table, the minimum value of  $n$ , given AQL,  $p_0$ , and  $\frac{L}{C}$ , was found. This was done by searching each column of the computer output until an  $x_2$ , rounded up, was found which was equal to the appropriate entry in Table V. These minimum values of  $n$  are noted in Table VI. By having these values in a table, it is possible to evaluate if any further consolidation is possible.

As can be seen for small values of AQL and small values of  $\frac{L}{C}$ , the values of  $n$  behave very well in that they change very little for different values of  $p_0$ . However, as either AQL or  $\frac{L}{C}$  approaches the upper limits under consideration, the values of  $n$  vary widely over the range of  $p_0$ .

TABLE V

Minimum Number of Defects in a Sample of Size n,  
Given an Acceptable Quality Level, to Meet an  $\alpha$  of 0.05

AQL	<u>.03</u>	<u>.05</u>	<u>.08</u>	<u>.10</u>	<u>.15</u>	<u>.20</u>	<u>.25</u>	<u>.30</u>
<u>n</u>								
10	2	3	3	4	4	5	6	6
20	3	4	5	5	7	8	9	10
30	4	5	6	7	9	11	12	14
40	4	5	7	8	11	13	16	18
50	5	6	8	10	13	16	19	21
60	5	7	9	11	15	18	22	25
70	6	8	11	12	17	21	25	28
80	6	8	12	14	18	23	27	32
90	7	9	13	15	20	25	30	35
100	7	10	14	16	22	28	33	39
110	7	10	15	17	24	30	36	42
120	8	11	16	19	26	32	39	45
130	8	12	17	20	27	35	42	49
140	9	12	18	21	29	37	45	52
150	9	13	19	22	31	39	47	55

TABLE VI  
Minimum Sample Size for a Given AQL,  $p_0$ , and  $\frac{L}{C}$  Ratio at  $\alpha = .05$

<u>AQL</u>	<u><math>p_0</math></u>	<u><math>\frac{L}{C}</math></u>	<u>.90</u>	<u>.91</u>	<u>.92</u>	<u>.93</u>	<u>.94</u>	<u>.95</u>	<u>.96</u>	<u>.97</u>
.03	.5		10	10	10	10	40	40	80	110
	.6		10	10	10	40	40	40	80	110
	.7		10	10	10	40	40	40	80	110
	.8		10	10	10	40	40	40	80	110
	.9		10	10	10	40	40	40	80	110
.05	.5		30	40	40	40	40	40	80	110
	.6		30	40	40	40	40	80	80	140
	.7		30	40	40	40	40	80	80	140
	.8		30	40	40	40	40	80	80	140
	.9		30	40	40	40	40	80	80	>150
.08	.5		30	30	40	40	50	60	110	150
	.6		30	40	40	50	50	100	130	>150
	.7		30	40	40	50	60	110	150	>150
	.8		30	40	40	50	60	120	150	>150
	.9		30	40	40	50	60	120	150	>150
.10	.5		20	40	40	40	60	70	70	110
	.6		40	40	40	60	70	100	110	>150
	.7		40	40	40	60	70	100	150	>150
	.8		40	40	60	70	90	110	>150	>150
	.9		40	40	60	70	100	110	>150	>150
.15	.6		40	50	50	60	80	80	80	130
	.7		50	60	70	80	80	130	130	>150
	.8		50	70	80	80	130	130	>150	>150
	.9		60	80	80	90	130	>150	>150	>150

TABLE VI (Continued)

<u>AQL</u>	<u>P<sub>0</sub></u>	<u><math>\frac{L}{C}</math></u>	<u>.90</u>	<u>.91</u>	<u>.92</u>	<u>.93</u>	<u>.94</u>	<u>.95</u>	<u>.96</u>	<u>.97</u>
.20	.6		40	40	40	40	40	40	60	60
	.7		40	60	60	60	80	90	110	120
	.8		60	60	80	90	120	150	>150	>150
	.9		60	80	90	120	>150	>150	>150	>150
.25	.7		30	30	50	60	70	70	80	80
	.8		70	80	80	80	100	140	150	>150
	.9		80	80	110	150	>150	>150	>150	>150
.30	.8		50	50	70	70	90	90	110	120
	.9		90	110	120	150	>150	>150	>150	>150

TABLE VII

Minimum Sample Size Given  $p_0$   
 And the  $\frac{L}{C}$  Ratio for AQL's From 0.03 - 0.30

$\frac{L}{C}$	$p_0$	<u>.5</u>	<u>.6</u>	<u>.7</u>	<u>.8</u>	<u>.9</u>
.90		20	30	30	50	50
.91		30	30	30	50	50
.92		30	30	40	60	80
.93		30	50	50	70	110
.94		50	50	70	70	110
.95		60	80	80	90	150
.96		80	90	100	140	(1)
.97		110	140	150	(1)	(1)

(1) Minimum sample size exceeds 150.

If AQL's of .10 or less are assumed to be the area of interest, it is apparent that a simple table using AQL and  $\frac{L}{C}$  as references could be used to determine a "good" value of  $n$ . However, when the wider range of AQL's (.03 - .30) was considered, it appeared more useful to create a table based on cross referencing  $p_0$  and  $\frac{L}{C}$ . Table VII was developed using the criteria that the sample size should be minimized subject to producer's risk being kept between 0.01 and 0.10. This range can be compared to producer's risk in Military Standard 105D which varies from .01 to .11. This objective was achieved with the exception of the starred entries in Table VII where sample sizes larger than 150 would have been required.

Using Table VII in conjunction with Table IV, it is possible to find a sampling plan for any of the identified AQL and  $p_0$  values. Having found the desired plan, the last item of interest would be to see what kind of operating characteristic curves the plan would have.

#### C. OPERATING CHARACTERISTIC CURVES

It is of interest to show how payments will vary over the range of  $p$  rather than at just two points, the AQL and  $p_0$ . To satisfy this need, operating characteristic curves may be developed. Having established a plan, we know sample size and the indifference function,  $h(p)$ . It is now

possible to calculate the expected price per satisfactory item,  $C(p)$ ,

$$C(p) = \frac{E[P_t]}{N(1-p)} = \frac{C(1 + A_1p + A_2p^2)}{(1-p)} .$$

In addition to a plot of  $C(p)$ , a complete set of operating characteristic curves would include a plot of two other functions, the "lower,"  $L(p)$ , and "upper,"  $U(p)$ , price per satisfactory item. They correspond, respectively, to a chosen  $\alpha$  and  $\beta$ . Here,  $L(p)$  and  $U(p)$  are defined as

$$L(p) = \frac{C}{1-p} \frac{Q(k_1)}{1-p} ,$$

$$\text{and } U(p) = \frac{C}{1-p} \frac{Q(k_2)}{1-p} ,$$

where  $k_1$  and  $k_2$  are values of  $x$  which, for a given  $p$ , satisfy the limits set for producer's or consumer's risk. This concept is best clarified by an example.

#### D. AN EXAMPLE

Suppose that a government agency is contracting out for bus service and that it has been decided that an acceptable quality level would be that buses could be off schedule ten percent of the time, i.e.,  $AQL = .10$ . It has also been decided that if buses aren't on time eighty percent of the time, then the contractor should be paid nothing, i.e.,  $p_0 = 0.8$ . In order to attract bids, it is decided that the lower limit should be set at ninety-five percent of the

average price paid for a good item, i.e.,  $\frac{L}{C} = 0.95$ . Using this information, we find from Table IV that  $A_1 = -.922$  and  $A_2 = 0.410$  and, from Table VII,  $n = 90$ . From this we have

$$C(p) = C(1 - .922 p - .41 p^2) / (1-p) ,$$

$$L(p) = C \left( 1 - \frac{.922}{90} k_1 + \frac{.41}{90(89)} k_1 - \frac{.41}{90(89)} k_1^2 \right) / (1-p) ,$$

$$\text{and } U(p) = C \left( 1 - \frac{.922}{90} k_2 + \frac{.41}{90(89)} k_2 - \frac{.41}{90(89)} k_2^2 \right) / (1-p) .$$

These equations are solved for various values of  $p$  and the resultant functional values are given in Table VIII. To find any  $k$  value, a cumulative binomial probability table can be used and an  $x$  can be found which satisfies the risk requirements. If  $p = 0.03$ ,  $n = 90$ , and  $\alpha = 0.05$ , then  $k_1 = 7$ ; if  $\alpha = 0.005$ , then  $k_1$  would be 9. As can be seen from Table VIII, the possibility of wide variation from  $C(p)$  increases dramatically as  $p$  increases. Another feature, which the table points out, is that negative payments are possible. This is not surprising, since the customer in identifying  $p_0$  has established the point beyond which the supplier's poor performance actually results in damages to the customer beyond the value of whatever good items have been received in the lot.

TABLE VIII  
Operating Characteristic Curve Values

<u>p</u>	<u>k<sub>1</sub><sup>(1)</sup></u>	<u>k<sub>2</sub><sup>(2)</sup></u>	<u>L(p)</u>	<u>C(p)</u>	<u>U(p)</u>
.03	7	1	.955	1.002	1.020
.05	9	2	.952	1.003	1.031
.08	13	4	.934	1.004	1.042
.10	15	5	.928	1.004	1.057
.15	20	9	.913	1.003	1.064
.20	25	13	.891	.999	1.074
.25	30	17	.864	.992	1.083
.30	35	21	.829	.981	1.091
.40	45	30	.729	.943	1.080
.50	54	39	.601	.873	1.049
.60	63	41	.387	.748	.982
.70	71	57	.061	.512	.842
.80	79	67	- .624	.000	.436
.90	86	77	-2.552	-1.619	- .883

(1)  $k_1$  figured at  $\alpha = .05$ .

(2)  $k_2$  figured at  $\beta = .10$ .

## VI. CONCLUSIONS AND RECOMMENDATIONS

In summary, the basic quantities needed to complete a PASS plan, with quadratic indifference, are  $n$ , the sample size, and  $h(p)$ , the indifference function. By using the idea suggested by Military Standard 105D of maximizing the price paid per nondefective item at the AQL, by noting the restrictions on the coefficients of the indifference function, and by defining  $h(p)$  to be 0 at  $p_0$ , it was possible to develop the proposed system which determined a set of indifference functions based on the values of AQL and  $p_0$ . Then a simplified method of finding sample size was presented which kept producer's risk between one and ten percent. Finally, the method for determining the proposed system's Operating Characteristics Curves was illustrated.

In order to implement the proposed system, only AQL,  $p_0$ , and the  $\frac{L}{C}$  ratio need to be known. With this information  $n$  and  $P_t$ , the payment function, can be found. After inspecting  $n$  items from a lot of size  $N$ , the number of defective items,  $x$ , will be known and the value  $P_t$  can be paid to the supplier.

The primary purpose of applying PASS, and also the proposed system, is to eliminate the necessity of rejecting lots. This is of particular advantage when the customer would rather not reject the lot, in the case of a badly needed

part, or cannot reject the lot, in the case of having services performed. The added advantage of the proposed system is that it eliminates the problem of identifying the indifference function and it shortens the time required to find  $n$ .

#### A. RECOMMENDATIONS FOR FURTHER STUDY

The following are recommended areas for further study:

1. It may be possible to develop a set of switching rules for the proposed system, similar to those in Military Standard 105D.
2. An area consciously omitted from this study was the problem of how to determine  $NC$ , the total price paid if  $p = 0$ .
3. In all probability the actual operating region for a supplier should be in the vicinity of the AQL. It may be possible to use a linear approximation to the indifference function in that region.
4. It may be enlightening to determine what effect increasing sample size (for a given indifference function) would have on the spread between  $U(p)$  and  $L(p)$ .
5. Betty Flehinger and James Miller [3] have done a paper on price paid being dependent on sampling; however, the detailed analysis is considered for repairable systems. The scheme proposed in this paper is more oriented toward non-repairable systems.

An analysis along the lines of ref. 3 for non-repairable systems, keeping PASS in mind could be interesting.

It is hoped that this work will not only be useful to those faced with the problem of being required to accept lots and needing a means to adjust price, but will also generate interest in further studies in this area.

# APPENDIX A Solutions to Equation (21)

AQL=0.03		P0=0.50			
N	L/C	0.90	0.91	0.92	0.93
10		3	3	1	1
20		4	4	2	2
30		6	4	3	3
40		7	5	4	4
50		8	5	5	5
60		9	6	6	6
70		10	7	7	7
80		11	8	8	8
90		12	9	9	9
100		13	10	10	10
110		14	11	11	11
120		15	12	12	12
130		16	13	13	13
140		17	14	14	14
150			15	15	15
		0.94	0.95	0.96	0.97
		0	1	1	1
		1	2	2	2
		2	3	3	3
		3	4	4	4
		4	5	5	5
		5	6	6	6
		6	7	7	7
		7	8	8	8
		8	9	9	9
		9	10	10	10
		10	11	11	11
		11	12	12	12
		12	13	13	13
		13	14	14	14
		14	15	15	15
		15	16	16	16
		16	17	17	17
		17	18	18	18
		18	19	19	19
		19	20	20	20
		20	21	21	21
		21	22	22	22
		22	23	23	23
		23	24	24	24
		24	25	25	25
		25	26	26	26
		26	27	27	27
		27	28	28	28
		28	29	29	29
		29	30	30	30
		30	31	31	31
		31	32	32	32
		32	33	33	33
		33	34	34	34
		34	35	35	35
		35	36	36	36
		36	37	37	37
		37	38	38	38
		38	39	39	39
		39	40	40	40
		40	41	41	41
		41	42	42	42
		42	43	43	43
		43	44	44	44
		44	45	45	45
		45	46	46	46
		46	47	47	47
		47	48	48	48
		48	49	49	49
		49	50	50	50
		50	51	51	51
		51	52	52	52
		52	53	53	53
		53	54	54	54
		54	55	55	55
		55	56	56	56
		56	57	57	57
		57	58	58	58
		58	59	59	59
		59	60	60	60
		60	61	61	61
		61	62	62	62
		62	63	63	63
		63	64	64	64
		64	65	65	65
		65	66	66	66
		66	67	67	67
		67	68	68	68
		68	69	69	69
		69	70	70	70
		70	71	71	71
		71	72	72	72
		72	73	73	73
		73	74	74	74
		74	75	75	75
		75	76	76	76
		76	77	77	77
		77	78	78	78
		78	79	79	79
		79	80	80	80
		80	81	81	81
		81	82	82	82
		82	83	83	83
		83	84	84	84
		84	85	85	85
		85	86	86	86
		86	87	87	87
		87	88	88	88
		88	89	89	89
		89	90	90	90
		90	91	91	91
		91	92	92	92
		92	93	93	93
		93	94	94	94
		94	95	95	95
		95	96	96	96
		96	97	97	97
		97	98	98	98
		98	99	99	99
		99	100	100	100
		100	101	101	101
		101	102	102	102
		102	103	103	103
		103	104	104	104
		104	105	105	105
		105	106	106	106
		106	107	107	107
		107	108	108	108
		108	109	109	109
		109	110	110	110
		110	111	111	111
		111	112	112	112
		112	113	113	113
		113	114	114	114
		114	115	115	115
		115	116	116	116
		116	117	117	117
		117	118	118	118
		118	119	119	119
		119	120	120	120
		120	121	121	121
		121	122	122	122
		122	123	123	123
		123	124	124	124
		124	125	125	125
		125	126	126	126
		126	127	127	127
		127	128	128	128
		128	129	129	129
		129	130	130	130
		130	131	131	131
		131	132	132	132
		132	133	133	133
		133	134	134	134
		134	135	135	135
		135	136	136	136
		136	137	137	137
		137	138	138	138
		138	139	139	139
		139	140	140	140
		140	141	141	141
		141	142	142	142
		142	143	143	143
		143	144	144	144
		144	145	145	145
		145	146	146	146
		146	147	147	147
		147	148	148	148
		148	149	149	149
		149	150	150	150

AQL=0.03		P0=0.60											
N	L/C	0.90	0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99	1.00	
10		3	2	1	0	0	0	0	0	0	0	0	
20		5	3	2	1	1	1	1	1	1	1	1	
30		7	4	3	2	2	2	2	2	2	2	2	
40		8	5	4	3	3	3	3	3	3	3	3	
50		9	6	5	4	4	4	4	4	4	4	4	
60		10	7	6	5	5	5	5	5	5	5	5	
70		12	8	7	6	6	6	6	6	6	6	6	
80		14	10	9	8	7	7	7	7	7	7	7	
90		16	12	11	10	9	9	9	9	9	9	9	
100		18	14	13	12	11	11	11	11	11	11	11	
110		20	16	15	14	13	13	13	13	13	13	13	
120		22	18	17	16	15	15	15	15	15	15	15	
130		24	20	19	18	17	17	17	17	17	17	17	
140		26	22	21	20	19	19	19	19	19	19	19	
150		28	24	23	22	21	21	21	21	21	21	21	

AQL=0.03		P0=0.70										
N	L/C	0.90	0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99	1.00
10		1.3	1.2	1.1	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3
20		2.3	2.2	2.1	2.0	1.9	1.8	1.7	1.6	1.5	1.4	1.3
30		3.3	3.2	3.1	3.0	2.9	2.8	2.7	2.6	2.5	2.4	2.3
40		4.3	4.2	4.1	4.0	3.9	3.8	3.7	3.6	3.5	3.4	3.3
50		5.3	5.2	5.1	5.0	4.9	4.8	4.7	4.6	4.5	4.4	4.3
60		6.3	6.2	6.1	6.0	5.9	5.8	5.7	5.6	5.5	5.4	5.3
70		7.3	7.2	7.1	7.0	6.9	6.8	6.7	6.6	6.5	6.4	6.3
80		8.3	8.2	8.1	8.0	7.9	7.8	7.7	7.6	7.5	7.4	7.3
90		9.3	9.2	9.1	9.0	8.9	8.8	8.7	8.6	8.5	8.4	8.3
100		10.3	10.2	10.1	10.0	9.9	9.8	9.7	9.6	9.5	9.4	9.3
110		11.3	11.2	11.1	11.0	10.9	10.8	10.7	10.6	10.5	10.4	10.3
120		12.3	12.2	12.1	12.0	11.9	11.8	11.7	11.6	11.5	11.4	11.3
130		13.3	13.2	13.1	13.0	12.9	12.8	12.7	12.6	12.5	12.4	12.3
140		14.3	14.2	14.1	14.0	13.9	13.8	13.7	13.6	13.5	13.4	13.3
150		15.3	15.2	15.1	15.0	14.9	14.8	14.7	14.6	14.5	14.4	14.3

AQL=0.03		P0=0.80										
N	L/C	0.90	0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99	1.00
10		3	1	1	0	0	0	0	0	0	0	0
20		5	2	2	1	1	1	1	1	1	1	1
30		8	3	3	2	2	2	2	2	2	2	2
40		0	4	4	3	3	3	3	3	3	3	3
50		2	5	5	4	4	4	4	4	4	4	4
60		5	8	6	5	5	5	5	5	5	5	5
70		7	9	7	6	6	6	6	6	6	6	6
80		0	1	8	7	7	7	7	7	7	7	7
90		0	2	9	8	8	8	8	8	8	8	8
100		2	4	0	9	9	9	9	9	9	9	9
110		5	7	6	0	0	0	0	0	0	0	0
120		7	9	0	1	1	1	1	1	1	1	1
130		9	0	1	2	2	2	2	2	2	2	2
140		2	1	2	3	3	3	3	3	3	3	3
150		4	3	3	4	4	4	4	4	4	4	4

AQL=0.03		P0=0.90	
N	L/C	0.90	0.91
10		3	2
20		12	12
30		23	23
40		35	35
50		47	47
60		58	58
70		69	69
80		79	79
90		89	89
100		99	99
110		109	109
120		119	119
130		129	129
140		139	139
150		149	149
	0.90	159	159
	0.91	169	169
	0.92	179	179
	0.93	189	189
	0.94	199	199
	0.95	209	209
	0.96	219	219
	0.97	229	229
	0.98	239	239
	0.99	249	249
	1.00	259	259

[illegible]

AQL=0.05		P0=0.60										
N	L/C	0.90	0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99	1.00
10												0.6
20			1.4	1.2	1.2	1.2	1.1	1.1	0.9	0.8	0.7	0.6
30		1.5	1.2	1.3	1.2	1.2	1.2	1.2	1.2	1.0	0.9	0.8
40		1.9	1.4	1.5	1.4	1.3	1.3	1.3	1.3	1.1	1.0	0.9
50		2.3	1.6	1.7	1.6	1.5	1.5	1.5	1.4	1.2	1.1	1.0
60		2.7	1.9	2.0	1.8	1.7	1.7	1.7	1.6	1.4	1.3	1.1
70		3.0	2.2	2.3	2.0	1.9	1.9	1.9	1.8	1.5	1.4	1.2
80		3.3	2.5	2.6	2.2	2.1	2.1	2.1	2.0	1.7	1.6	1.3
90		3.6	2.8	2.9	2.4	2.3	2.3	2.3	2.2	1.9	1.8	1.4
100		3.9	3.1	3.2	2.6	2.5	2.5	2.5	2.4	2.1	2.0	1.5
110		4.2	3.4	3.5	2.8	2.7	2.7	2.7	2.6	2.2	2.1	1.6
120		4.5	3.7	3.8	3.0	2.9	2.9	2.9	2.8	2.3	2.2	1.7
130		4.8	4.0	4.1	3.2	3.1	3.1	3.1	3.0	2.4	2.3	1.8
140		5.0	4.3	4.4	3.4	3.3	3.3	3.3	3.2	2.5	2.4	1.9
150		5.3	4.6	4.7	3.6	3.5	3.5	3.5	3.4	2.6	2.5	2.0

AQL=0.05		P0=0.70									
N	L/C	0.90	0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.98	1.00
10		1.5	1.4	3.5	2.4	1.2	0.0	0.8	0.9	0.5	0.6
20		2.4	2.2	5.0	3.4	2.3	1.2	1.2	1.2	1.2	1.1
30		3.7	3.4	6.7	4.5	3.4	3.4	3.6	3.4	2.9	1.6
40		5.1	4.5	8.0	5.7	4.5	4.5	4.5	4.2	3.6	1.6
50		7.8	6.8	10.3	7.8	5.6	5.9	5.4	4.8	4.3	1.7
60		11.5	9.5	13.5	10.1	7.6	9.9	6.7	6.4	5.0	2.7
70		15.9	12.3	17.0	12.6	9.8	13.8	8.9	7.2	5.4	3.4
80		21.7	16.0	22.2	16.7	12.7	18.8	10.8	8.8	6.4	4.4
90		27.1	19.3	27.7	19.9	15.8	23.7	13.7	10.8	7.7	5.5
100		33.0	23.6	33.4	23.0	18.9	29.8	16.6	12.7	9.1	6.6
110		39.4	28.3	39.7	27.4	21.6	36.1	19.4	14.2	10.6	7.7
120		46.2	33.4	46.1	31.8	24.0	42.7	22.3	16.2	12.2	8.8
130		53.5	38.9	53.0	36.3	26.3	49.6	25.2	18.0	13.9	
140		61.2	44.8	60.4	41.0	28.6	56.8	28.1	20.0	15.6	
150		69.2	51.1	68.0	46.0	30.9	64.3	31.0	21.2	17.3	

AQL=0.05		P0=0.80									
N	L/C	0.90	0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.98	1.00
10		1.5	1.4	3.5	2.4	1.2	0.0	0.8	0.8	0.7	0.5
20		2.4	2.2	5.0	3.4	2.3	1.2	1.2	1.2	1.2	1.1
30		3.7	3.4	6.7	4.5	3.4	3.4	3.6	3.4	2.9	1.6
40		5.1	4.5	8.0	5.7	4.5	4.5	4.5	4.2	3.6	1.6
50		7.8	6.8	10.3	7.8	5.6	5.9	5.4	4.8	4.3	1.7
60		11.5	9.5	13.5	10.1	7.6	9.9	6.7	6.4	5.0	2.7
70		15.9	12.3	17.0	12.6	9.8	13.8	8.9	7.2	5.4	3.4
80		21.7	16.0	22.2	16.7	12.7	18.8	10.8	8.8	6.4	4.4
90		27.1	19.3	27.7	19.9	15.8	23.7	13.7	10.8	7.7	5.5
100		33.0	23.6	33.4	23.0	18.9	29.8	16.6	12.7	9.1	6.6
110		39.4	28.3	39.7	27.4	21.6	36.1	19.4	14.2	10.6	7.7
120		46.2	33.4	46.1	31.8	24.0	42.7	22.3	16.2	12.2	8.8
130		53.5	38.9	53.0	36.3	26.3	49.6	25.2	18.0	13.9	
140		61.2	44.8	60.4	41.0	28.6	56.8	28.1	20.0	15.6	
150		69.2	51.1	68.0	46.0	30.9	64.3	31.0	21.2	17.3	

AQL=0.05		PO=0.90											
N	L/C	0.90	0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99	1.00	
10													
20		1:5	1:4	3:5	1:2	1:2	0:0	0:8	0:6	0:7	0:6	0:5	
30		2:4	2:4	2:5	2:3	2:3	1:2	1:3	1:2	1:2	1:1	1:1	
40		3:8	3:4	3:5	3:4	3:4	2:3	2:4	2:3	2:3	2:2	2:2	
50		4:2	4:1	4:5	4:5	4:5	3:4	3:5	3:4	3:4	3:3	3:3	
60		5:7	5:8	5:8	5:8	5:8	4:5	4:6	4:5	4:5	4:4	4:4	
70		6:2	6:1	6:5	6:5	6:5	5:6	5:7	5:6	5:6	5:5	5:5	
80		7:1	7:4	7:8	7:8	7:8	6:7	6:8	6:7	6:7	6:6	6:6	
90		8:1	8:4	8:5	8:5	8:5	7:8	7:9	7:8	7:8	7:7	7:7	
100		9:0	9:4	9:6	9:6	9:6	8:9	8:9	8:8	8:8	8:7	8:7	
110		10:4	10:5	10:8	10:8	10:8	9:10	9:10	9:9	9:9	9:8	9:8	
120		11:3	11:5	11:6	11:6	11:6	10:11	10:11	10:10	10:10	10:9	10:9	
130		12:7	12:8	12:9	12:9	12:9	11:12	11:12	11:11	11:11	11:10	11:10	
140		13:2	13:5	13:6	13:6	13:6	12:13	12:13	12:12	12:12	12:11	12:11	
150		14:6	14:9	14:9	14:9	14:9	13:14	13:14	13:13	13:13	13:12	13:12	

AQL=0.08		P0=0.50											
N	L/C	0.90	0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99	1.00	
		2 3 5 8 10 13 15 18 20 22 23 25	1 3 5 6 8 9 11 13 14 16 17 19 20 22 24	1 3 4 6 7 9 11 12 14 15 17 18 20 21 23 24	1 3 4 6 7 9 10 12 13 15 16 18 19 21 22	1 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25	1 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25	1 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25	1 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25	1 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25	1 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25	1 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25	1 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25

AQL=0.08		P0=0.60										
N	L/C	0.90	0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99	1.00
		9 1.5 2.5 3.5 4.5 5.5 6.5 7.5 8.5 9.5 10.5 11.5 12.5 13.5 14.5 15.5	1 3.4 5.6 7.8 9.9 11.1 12.2 13.3 14.4 15.5 16.6 17.7 18.8 19.9 20.0	1 3.3 4.4 5.5 6.6 7.7 8.8 9.9 10.0 11.1 12.2 13.3 14.4 15.5 16.6 17.7	1 3.2 4.3 5.4 6.5 7.6 8.7 9.8 10.9 11.0 12.1 13.2 14.3 15.4 16.5 17.6	1 3.1 4.2 5.3 6.4 7.5 8.6 9.7 10.8 11.9 12.0 13.1 14.2 15.3 16.4 17.5 18.6	1 3.0 4.1 5.2 6.3 7.4 8.5 9.6 10.7 11.8 12.9 13.0 14.1 15.2 16.3 17.4 18.5 19.6 20.7	1 2.9 3.9 4.9 5.9 6.9 7.9 8.9 9.9 10.9 11.9 12.9 13.9 14.9 15.9 16.9 17.9 18.9 19.9 20.9	1 2.8 3.7 4.6 5.5 6.4 7.3 8.2 9.1 10.0 10.9 11.8 12.7 13.6 14.5 15.4 16.3 17.2 18.1 19.0 20.0	1 2.7 3.5 4.3 5.1 5.9 6.7 7.5 8.3 9.1 9.9 10.7 11.5 12.3 13.1 13.9 14.7 15.5 16.3 17.1 17.9 18.7 19.5 20.3	1 2.6 3.3 4.0 4.7 5.4 6.1 6.8 7.5 8.2 8.9 9.6 10.3 11.0 11.7 12.4 13.1 13.8 14.5 15.2 15.9 16.6 17.3 18.0 18.7 19.4 20.1	1 2.5 3.2 3.9 4.6 5.3 6.0 6.7 7.4 8.1 8.8 9.5 10.2 10.9 11.6 12.3 13.0 13.7 14.4 15.1 15.8 16.5 17.2 17.9 18.6 19.3 20.0

AQL=0.08		P0=0.70											
N	L/C	0.90	0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99	1.00	
10												0.8	
20		1.8	1.7	1.6	1.5	1.5	1.4	1.3	1.2	1.1	1.0	0.9	
30		5.2	4.9	4.6	4.5	4.4	4.2	4.0	3.8	3.6	3.4	3.2	
40		9.6	9.2	8.8	8.5	8.3	8.0	7.7	7.5	7.2	7.0	6.7	
50		13.0	12.6	12.2	11.8	11.5	11.2	10.9	10.6	10.3	10.1	9.8	
60		16.3	15.9	15.5	15.1	14.8	14.4	14.1	13.8	13.5	13.2	13.0	
70		19.7	19.3	18.9	18.5	18.2	17.8	17.5	17.2	16.9	16.6	16.3	
80		23.1	22.7	22.3	21.9	21.6	21.2	20.9	20.6	20.3	20.1	19.8	
90		26.4	26.0	25.6	25.2	24.9	24.5	24.2	23.9	23.6	23.4	23.1	
100		29.8	29.4	29.0	28.6	28.3	27.9	27.6	27.3	27.0	26.8	26.5	
110		33.1	32.7	32.3	31.9	31.6	31.2	30.9	30.6	30.3	30.1	29.8	
120		36.5	36.1	35.7	35.3	35.0	34.6	34.3	34.0	33.7	33.4	33.1	
130		39.8	39.4	39.0	38.6	38.3	37.9	37.6	37.3	37.0	36.8	36.5	
140		43.2	42.8	42.4	42.0	41.7	41.3	41.0	40.7	40.4	40.2	39.9	
150		46.5	46.1	45.7	45.3	45.0	44.6	44.3	44.0	43.7	43.4	43.1	

AQL=0.08		P0=0.80	
N	L/C	0.90	0.91
10		1.8	1.7
20		1.5	1.3
30		1.5	1.3
40		1.5	1.3
50		1.5	1.3
60		1.5	1.3
70		1.5	1.3
80		1.5	1.3
90		1.5	1.3
100		1.5	1.3
110		1.5	1.3
120		1.5	1.3
130		1.5	1.3
140		1.5	1.3
150		1.5	1.3
	0.90	1.8	1.7
	0.91	1.5	1.3
	0.92	1.5	1.3
	0.93	1.5	1.3
	0.94	1.5	1.3
	0.95	1.5	1.3
	0.96	1.5	1.3
	0.97	1.5	1.3
	0.98	1.5	1.3
	0.99	1.5	1.3
	1.00	1.5	1.3

AQL=0.08		P0=0.90											
N	L/C	0.90	0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99	1.00	
10		7	6	6	5	4	3	2	1	0	0	0	8
20		5	3	3	4	4	2	1	1	1	1	1	1
30		5	4	4	5	5	1	1	2	2	2	2	2
40		6	6	6	6	6	3	3	3	3	3	3	4
50		6	8	7	8	8	5	5	4	4	4	4	5
60		8	9	9	10	9	7	7	6	6	6	6	7
70		10	11	10	11	10	8	8	7	7	7	7	8
80		12	13	12	13	12	10	10	9	9	9	9	10
90		15	16	15	16	15	12	12	11	11	11	11	12
100		17	19	18	19	18	14	14	13	13	13	13	14
110		20	22	21	22	21	16	16	15	15	15	15	16
120		22	24	23	24	23	17	17	16	16	16	16	17
130		25											
140													
150													

AQL=0.10 P0=0.50

N	L/C	0.90	0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99	1.00
10		3	2	2	1	0	0	9	8	8	7	6
20		4	4	3	2	2	2	3	3	3	3	2
30		6	5	5	3	3	3	4	4	4	4	4
40		9	7	7	5	5	5	6	6	6	6	6
50		11	9	9	7	7	7	8	8	8	8	8
60		13	11	10	8	8	8	10	10	10	10	10
70		15	13	12	10	10	10	12	12	12	12	12
80		17	15	14	12	12	12	14	14	14	14	14
90		19	17	16	14	14	14	16	16	16	16	16
100		21	19	18	16	16	16	18	18	18	18	18
110		23	21	20	18	18	18	20	20	20	20	20
120		25	23	22	20	20	20	22	22	22	22	22
130		26	24	23	21	21	21	23	23	23	23	23
140		28	26	25	23	23	23	25	25	25	25	25
150												

AQL=0.10 P0=0.60

N	L/C	0.90	0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99	1.00
10		1	1	0	0	0	0	1	1	1	1	1
20		2	2	1	1	1	1	2	2	2	2	2
30		4	3	2	2	2	2	4	4	4	4	4
40		7	5	4	3	3	3	7	7	7	7	7
50		11	9	7	6	6	6	11	11	11	11	11
60		15	13	10	9	9	9	15	15	15	15	15
70		19	17	14	12	12	12	19	19	19	19	19
80		23	21	18	16	16	16	23	23	23	23	23
90		27	25	22	20	20	20	27	27	27	27	27
100		31	29	26	24	24	24	31	31	31	31	31
110		35	33	30	28	28	28	35	35	35	35	35
120		39	37	34	32	32	32	39	39	39	39	39
130		43	41	38	36	36	36	43	43	43	43	43
140		47	45	42	40	40	40	47	47	47	47	47
150		51	49	46	44	44	44	51	51	51	51	51

AQL=0.10		P0=0.70							
N	L/C	0.90	0.91	0.92	0.93	0.94	0.95	0.96	0.97
10									
20		2	3	4	5	6	7	8	9
30		3	4	5	6	7	8	9	10
40		4	5	6	7	8	9	10	11
50		5	6	7	8	9	10	11	12
60		6	7	8	9	10	11	12	13
70		7	8	9	10	11	12	13	14
80		8	9	10	11	12	13	14	15
90		9	10	11	12	13	14	15	16
100		10	11	12	13	14	15	16	17
110		11	12	13	14	15	16	17	18
120		12	13	14	15	16	17	18	19
130		13	14	15	16	17	18	19	20
140		14	15	16	17	18	19	20	21
150		15	16	17	18	19	20	21	22

AQL=0.10		P0=0.80							
N	L/C	0.90	0.91	0.92	0.93	0.94	0.95	0.96	0.97
10									
20		2	3	4	5	6	7	8	9
30		3	4	5	6	7	8	9	10
40		4	5	6	7	8	9	10	11
50		5	6	7	8	9	10	11	12
60		6	7	8	9	10	11	12	13
70		7	8	9	10	11	12	13	14
80		8	9	10	11	12	13	14	15
90		9	10	11	12	13	14	15	16
100		10	11	12	13	14	15	16	17
110		11	12	13	14	15	16	17	18
120		12	13	14	15	16	17	18	19
130		13	14	15	16	17	18	19	20
140		14	15	16	17	18	19	20	21
150		15	16	17	18	19	20	21	22

AQL=0.10		P0=0.90	
N	L/C	0.90	0.91
10		9	8
20		13	13
30		57	53
40		96	91
50		135	127
60		174	165
70		213	203
80		252	240
90		291	278
100		330	316
110		369	354
120		408	392
130		447	430
140		486	468
150		525	506
	0.90	135:213:291:369:447:525	102:168:234:300:366:432
	0.91	135:213:291:369:447:525	102:168:234:300:366:432
	0.92	135:213:291:369:447:525	102:168:234:300:366:432
	0.93	135:213:291:369:447:525	102:168:234:300:366:432
	0.94	135:213:291:369:447:525	102:168:234:300:366:432
	0.95	135:213:291:369:447:525	102:168:234:300:366:432
	0.96	135:213:291:369:447:525	102:168:234:300:366:432
	0.97	135:213:291:369:447:525	102:168:234:300:366:432
	0.98	135:213:291:369:447:525	102:168:234:300:366:432
	0.99	135:213:291:369:447:525	102:168:234:300:366:432
	1.00	135:213:291:369:447:525	102:168:234:300:366:432

AQL=0.15		P0=0.60			
N	L/C	0.90	0.91	0.92	0.93
10		2.8	7.2	7.0	6.9
20		5.3	2.6	4.8	4.7
30		7.8	0.5	7.8	7.9
40		10.3	0.9	9.2	9.5
50		12.7	1.4	11.5	11.8
60		15.2	1.9	14.0	14.3
70		17.7	2.3	16.5	16.8
80		20.2	2.8	19.0	19.3
90		22.7	3.2	21.5	21.8
100		25.2	3.7	24.0	24.3
110		27.7	4.1	26.5	26.8
120		30.2	4.6	29.0	29.3
130		32.7	5.0	31.5	31.8
140		35.2	5.5	34.0	34.3
150		37.7	6.0	36.5	36.8
	0.94	40.2	6.5	39.0	39.3
	0.95	42.7	7.0	41.5	41.8
	0.96	45.2	7.5	44.0	44.3
	0.97	47.7	8.0	46.5	46.8
	0.98	50.2	8.5	49.0	49.3
	0.99	52.7	9.0	51.5	51.8
	1.00	55.2	9.5	54.0	54.3

AQL=0.15		P0=0.70			
N	L/C	0.90	0.91	0.92	0.93
10		6.0	5.9	5.7	4.6
20		5.9	2.6	7.0	6.8
30		9.3	0.3	9.3	9.0
40		12.6	0.7	11.5	11.3
50		15.9	1.1	14.8	14.5
60		19.2	1.4	18.0	17.7
70		22.5	1.8	21.3	21.0
80		25.8	2.1	24.6	24.3
90		29.1	2.5	27.9	27.6
100		32.4	2.9	31.2	30.9
110		35.7	3.3	34.5	34.2
120		39.0	3.7	37.8	37.5
130		42.3	4.1	41.1	40.8
140		45.6	4.5	44.4	44.1
150		48.9	4.9	47.7	47.4
	0.94	52.2	5.3	51.0	50.7
	0.95	55.5	5.7	54.3	54.0
	0.96	58.8	6.1	57.6	57.3
	0.97	62.1	6.5	60.9	60.6
	0.98	65.4	6.9	64.2	63.9
	0.99	68.7	7.3	67.5	67.2
	1.00	72.0	7.7	70.8	70.5

AQL=0.15 P0=0.80

N	L/C	0.90	0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99	1.00
10		5	4	3	2	2	1	0	9	8	7	7
20		4	7	6	4	4	4	3	5	5	4	3
30		7	9	8	6	8	6	7	7	6	5	4
40		9	0	9	8	9	7	9	9	8	4	0
50		1	1	1	1	1	1	1	1	1	1	0
60		2	2	2	2	2	2	2	2	2	2	2
70		4	3	3	3	3	3	3	3	3	3	3
80		8	6	5	5	6	5	5	6	5	5	4
90		2	6	0	2	7	0	4	8	6	2	0
100		1	1	1	1	1	1	1	1	1	1	1
110		2	2	2	2	2	2	2	2	2	2	2
120		4	3	3	3	3	3	3	3	3	3	3
130		8	6	5	5	6	5	5	6	5	5	4
140		2	6	0	2	7	0	4	8	6	2	0
150		1	1	1	1	1	1	1	1	1	1	1

AQL=0.15 P0=0.90

N	L/C	0.90	0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99	1.00
10		4	3	2	2	2	1	0	9	8	7	6
20		7	6	6	6	6	5	4	5	5	4	3
30		9	9	8	8	8	7	6	7	6	5	4
40		1	1	1	1	1	1	1	1	1	1	0
50		2	2	2	2	2	2	2	2	2	2	2
60		4	3	3	3	3	3	3	3	3	3	3
70		8	6	5	5	6	5	5	6	5	5	4
80		2	6	0	2	7	0	4	8	6	2	0
90		1	1	1	1	1	1	1	1	1	1	1
100		2	2	2	2	2	2	2	2	2	2	2
110		4	3	3	3	3	3	3	3	3	3	3
120		8	6	5	5	6	5	5	6	5	5	4
130		2	6	0	2	7	0	4	8	6	2	0
140		1	1	1	1	1	1	1	1	1	1	1
150		2	2	2	2	2	2	2	2	2	2	2

[illegible]

AQL=0.20	P0=0.70
N	L/C
10	0.90
20	0.91
30	0.92
40	0.93
50	0.94
60	0.95
70	0.96
80	0.97
90	0.98
100	0.99
110	1.00
120	
130	
140	
150	

AQL=0.20		P0=0.80											
N	L/C	0.90	0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99	1.00	
10		3.0	3.08	3.14	3.2	3.25	3.3	3.35	3.4	3.45	3.5	3.55	
20		3.5	3.58	3.64	3.7	3.75	3.8	3.85	3.9	3.95	4.0	4.05	
30		3.9	3.98	4.04	4.1	4.15	4.2	4.25	4.3	4.35	4.4	4.45	
40		4.3	4.38	4.44	4.5	4.55	4.6	4.65	4.7	4.75	4.8	4.85	
50		4.7	4.78	4.84	4.9	4.95	5.0	5.05	5.1	5.15	5.2	5.25	
60		5.1	5.18	5.24	5.3	5.35	5.4	5.45	5.5	5.55	5.6	5.65	
70		5.5	5.58	5.64	5.7	5.75	5.8	5.85	5.9	5.95	6.0	6.05	
80		5.9	5.98	6.04	6.1	6.15	6.2	6.25	6.3	6.35	6.4	6.45	
90		6.3	6.38	6.44	6.5	6.55	6.6	6.65	6.7	6.75	6.8	6.85	
100		6.7	6.78	6.84	6.9	6.95	7.0	7.05	7.1	7.15	7.2	7.25	
110		7.1	7.18	7.24	7.3	7.35	7.4	7.45	7.5	7.55	7.6	7.65	
120		7.5	7.58	7.64	7.7	7.75	7.8	7.85	7.9	7.95	8.0	8.05	
130		7.9	7.98	8.04	8.1	8.15	8.2	8.25	8.3	8.35	8.4	8.45	
140		8.3	8.38	8.44	8.5	8.55	8.6	8.65	8.7	8.75	8.8	8.85	
150		8.7	8.78	8.84	8.9	8.95	9.0	9.05	9.1	9.15	9.2	9.25	

AQL=0.20		P0=0.90											
N	L/C	0.90	0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99	1.00	
10		9.7	8.9	7.1	7.3	6.1	5.0	4.8	3.9	3.5	2.3	1.2	
20		11.4	10.3	8.5	10.5	9.2	7.9	7.2	6.0	5.7	4.6	3.4	
30		13.1	11.8	10.0	12.0	10.5	9.3	8.6	7.3	6.9	5.8	4.5	
40		14.7	13.3	11.4	13.5	11.9	10.7	9.9	8.5	8.1	6.9	5.6	
50		16.4	14.8	12.8	15.0	13.3	12.1	11.2	9.7	9.3	8.0	6.7	
60		18.1	16.3	14.2	16.5	14.7	13.5	12.5	10.9	10.5	9.1	7.8	
70		19.7	17.8	15.6	18.0	16.1	14.9	13.8	12.1	11.7	10.2	8.9	
80		21.4	19.3	17.1	19.5	17.5	16.3	15.2	13.4	13.0	11.4	10.0	
90		23.1	21.0	18.7	21.0	19.0	17.7	16.6	14.7	14.3	12.6	11.1	
100		24.7	22.5	20.2	22.5	20.4	19.1	18.0	16.1	15.7	13.9	12.4	
110		26.4	24.1	21.8	24.0	21.9	20.5	19.4	17.5	17.1	15.2	13.6	
120		28.1	25.7	23.4	25.5	23.3	21.9	20.8	18.9	18.5	16.5	14.9	
130		29.7	27.2	25.0	27.0	24.7	23.3	22.2	20.3	19.9	17.8	16.1	
140		31.4	28.8	26.5	28.5	26.2	24.7	23.6	21.7	21.3	19.1	17.4	
150		33.1	30.3	28.0	30.0	27.7	26.2	25.1	23.2	22.8	20.5	18.8	

AQL=0.25 P0=0.70

N	L/C	0.90	0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99	1.00
10		4.0	3.5	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0
20		7.0	6.0	5.0	4.0	3.0	2.0	1.0	0.0	0.0	0.0	0.0
30		11.0	9.0	7.0	5.0	3.0	2.0	1.0	0.0	0.0	0.0	0.0
40		14.0	11.0	8.0	5.0	3.0	2.0	1.0	0.0	0.0	0.0	0.0
50		16.0	13.0	9.0	6.0	4.0	3.0	2.0	1.0	0.0	0.0	0.0
60		18.0	14.0	10.0	7.0	5.0	4.0	3.0	2.0	1.0	0.0	0.0
70		20.0	15.0	11.0	8.0	6.0	5.0	4.0	3.0	2.0	1.0	0.0
80		22.0	16.0	12.0	9.0	7.0	6.0	5.0	4.0	3.0	2.0	1.0
90		24.0	17.0	13.0	10.0	8.0	7.0	6.0	5.0	4.0	3.0	2.0
100		26.0	18.0	14.0	11.0	9.0	8.0	7.0	6.0	5.0	4.0	3.0
110		28.0	19.0	15.0	12.0	10.0	9.0	8.0	7.0	6.0	5.0	4.0
120		30.0	20.0	16.0	13.0	11.0	10.0	9.0	8.0	7.0	6.0	5.0
130		32.0	21.0	17.0	14.0	12.0	11.0	10.0	9.0	8.0	7.0	6.0
140		34.0	22.0	18.0	15.0	13.0	12.0	11.0	10.0	9.0	8.0	7.0
150		36.0	23.0	19.0	16.0	14.0	13.0	12.0	11.0	10.0	9.0	8.0

AQL=0.25 P0=0.80

N	L/C	0.90	0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99	1.00
10		6.0	5.0	4.0	3.0	2.0	1.0	0.0	0.0	0.0	0.0	0.0
20		10.0	8.0	6.0	4.0	2.0	1.0	0.0	0.0	0.0	0.0	0.0
30		14.0	11.0	8.0	5.0	3.0	2.0	1.0	0.0	0.0	0.0	0.0
40		17.0	13.0	9.0	6.0	4.0	3.0	2.0	1.0	0.0	0.0	0.0
50		20.0	15.0	10.0	7.0	5.0	4.0	3.0	2.0	1.0	0.0	0.0
60		23.0	17.0	11.0	8.0	6.0	5.0	4.0	3.0	2.0	1.0	0.0
70		26.0	19.0	12.0	9.0	7.0	6.0	5.0	4.0	3.0	2.0	1.0
80		29.0	21.0	13.0	10.0	8.0	7.0	6.0	5.0	4.0	3.0	2.0
90		32.0	23.0	14.0	11.0	9.0	8.0	7.0	6.0	5.0	4.0	3.0
100		35.0	25.0	15.0	12.0	10.0	9.0	8.0	7.0	6.0	5.0	4.0
110		38.0	27.0	16.0	13.0	11.0	10.0	9.0	8.0	7.0	6.0	5.0
120		41.0	29.0	17.0	14.0	12.0	11.0	10.0	9.0	8.0	7.0	6.0
130		44.0	31.0	18.0	15.0	13.0	12.0	11.0	10.0	9.0	8.0	7.0
140		47.0	33.0	19.0	16.0	14.0	13.0	12.0	11.0	10.0	9.0	8.0
150		50.0	35.0	20.0	17.0	15.0	14.0	13.0	12.0	11.0	10.0	9.0

AQL=0.25		P0=0.90											
N	L/C	0.90	0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99	1.00	
10		3.7	3.6	3.4	3.3	3.1	3.0	0.9	9.7	8.6	7.4	7.3	
20		6.1	5.8	5.6	5.4	5.2	5.0	8.7	8.5	8.3	8.1	8.0	
30		7.1	6.8	6.6	6.4	6.2	6.0	9.1	8.9	8.7	8.5	8.4	
40		7.7	7.4	7.2	7.0	6.8	6.6	9.4	9.2	9.0	8.8	8.7	
50		8.1	7.8	7.6	7.4	7.2	7.0	9.6	9.4	9.2	9.0	8.9	
60		8.4	8.1	7.9	7.7	7.5	7.3	9.8	9.6	9.4	9.2	9.1	
70		8.7	8.4	8.2	8.0	7.8	7.6	10.0	9.8	9.6	9.4	9.3	
80		8.9	8.6	8.4	8.2	8.0	7.8	10.2	10.0	9.8	9.6	9.5	
90		9.1	8.8	8.6	8.4	8.2	8.0	10.4	10.2	10.0	9.8	9.7	
100		9.3	9.0	8.8	8.6	8.4	8.2	10.6	10.4	10.2	10.0	9.9	
110		9.5	9.2	9.0	8.8	8.6	8.4	10.8	10.6	10.4	10.2	10.1	
120		9.7	9.4	9.2	9.0	8.8	8.6	11.0	10.8	10.6	10.4	10.3	
130		9.9	9.6	9.4	9.2	9.0	8.8	11.2	11.0	10.8	10.6	10.5	
140		10.1	9.8	9.6	9.4	9.2	9.0	11.4	11.2	11.0	10.8	10.7	
150		10.3	10.0	9.8	9.6	9.4	9.2	11.6	11.4	11.2	11.0	10.9	

[illegible][illegible]

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